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AN ANALYTICAL SOLUTION OF THE HEAT FLOW IN A GUN TUBE

TECHNICAL REPORT

Dr. Shih-Chi Chu
and
Philip D. Benzkofer

November 1970

SCIENCE & TECHNOLOGY LABORATORY

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U. S. ARMY WEAPONS COMMAND

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ABSTRACT

The analytical solution of the heat conduction equation for a gun tube (circular, thick-walled cylinder) assumed, for simplicity, to be of infinite length, is obtained. The temperature function is assumed to have axial and angular symmetry and, therefore, is dependent only upon time and radial distance. The axial flow has been neglected because the projectile moves through the tube at a much faster rate than heat can penetrate the tube. The initial and boundary conditions have been considered as close as possible to those encountered during the repeated firing of a gun. The temperature distribution in a barrel can be determined provided that the propellant gas temperature during the firing of each round is known and that the nature of the heat transfer from the gas to the barrel wall is known.

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NOMENCLATURE

a = inside radius of gun tube

b = outside radius of gun tube

t = time

h_1, h_2 = gas and natural convection coefficients of gun tube, respectively

K = thermal conductivity of gun tube

r = radial distance of a point from the axis of the cylinder

$T(r,t)$ = temperature of point r at time t

$T_g(t)$ = propellant gas temperature function

$T_a(r,t)$ = a transient part of temperature of the total temperature

$T_b(r,t)$ = a transient part of temperature of the total temperature

$T_s(r)$ = a steady-state part of temperature of the total temperature

T_A = ambient temperature

α = thermal diffusivity of barrel material

β_n = eigenvalues of equation (30)

J_0, Y_0 = Bessel functions of first and second kinds of order zero, respectively

J_1, Y_1 = Bessel functions of first and second kinds of order one, respectively

I_0, K_0 = modified Bessel functions of first and second kinds of order zero, respectively

I_1, K_1 = modified Bessel functions of first and second kinds of order one, respectively

$f(r)$ = initial temperature distribution

INTRODUCTION

The solution to the initial-boundary value problem for the conduction of heat in a thick-walled cylinder is of considerable technological importance, particularly, in the gun tube heat transfer analysis. These problems have consequently attracted considerable attention, and a number of special solutions[1-6] have been developed. Mow, Pascual, and Pascale[1] dealt with the problem of radial flow of heat in a gun tube assumed to be infinitely long. In their investigations, the initial and boundary conditions were assumed to be $T(r,t) = 0$ for $t \leq 0$, $T(a,t) = \phi(t)$ for $t \geq 0$, and $T(b,t) = 0$ for all t. Apparently, the surface conductance on both inner and outer surfaces of a gun tube has been disregarded in their analysis. A similar problem, with a convective boundary condition on the outer surface of a gun tube, was solved by Pascual, Zweig, and Sutherland.[2] By virtue of the fact that the initial condition was assumed to be zero in the above two investigations, the problem solved by these investigators is essentially the first round problem. Comenetz[3] gave a solution for the temperature distribution in a hollow cylinder of infinite length supplied with heat through the inner surface. The rate of heat input was permitted to vary linearly (or at most quadratically) with time and the initial temperature of the hollow cylinder was assumed to be zero. A more general solution of heat flow in a finite, hollow, circular cylinder was given by Ölcer.[4] However, Ölcer's method is very time-consuming and difficult to obtain desirable numerical results. To solve the gun barrel heat-transfer problem practically, a relatively simple solution that closely approximates the actual firing conditions of the gun is necessary. This report is intended to serve this purpose.

OBJECTIVE

One of the many causes of gun barrel failure occurs when the effective thermal stress level in the interior of the small caliber gun tube exceeds the yield strength of the barrel material.

To determine the thermal stresses in a barrel, the temperature distribution within a barrel must first be determined. Therefore, the purpose of this investigation is to obtain the analytical solution of the heat conduction equation for a gun tube subjected to an arbitrary initial condition and to boundary conditions that are as close as possible to those encountered during the actual single and repeated firing.

FORMULATION OF THE PROBLEM

The unsteady temperature field in a homogeneous, isotropic gun tube is assumed to have axial and angular symmetry and, therefore, is dependent only upon time and radial distance. For simplicity, the gun tube is assumed to be of infinite length, and the thermal properties assumed to be independent of the temperature for each small time-interval during each firing round. Then the heat flow in a gun tube is governed by the well-known Fourier heat conduction equation.[5]

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{1}{\alpha}\frac{\partial T}{\partial t}; \qquad a < r < b,\ t > 0 \tag{1}$$

where $\alpha > 0$ denotes thermal diffusivity. In addition, the following initial and boundary conditions are specified for $T(r,t)$:

$$T(r,0) = f(r); \qquad a < r < b,\ t = 0 \tag{2}$$

$$K\frac{\partial T}{\partial r} = -h_1[T_g(t)-T]; \qquad r = a,\ t > 0 \tag{3}$$

$$K\frac{\partial T}{\partial r} = -h_2[T-T_A]; \qquad r = b,\ t > 0 \tag{4}$$

where $T_g(t)$ is the propellant gas temperature function at the bore surface, T_A is the ambient temperature, $f(r)$ is the initial temperature distribution at the beginning of each small time-increment, or the beginning of each round, h_1 and h_2 are respectively gas and natural convection coefficients, and K is the thermal conductivity of the gun tube.

The convection coefficients, h_1 and h_2, at the bore surface and at the outer surface of a barrel may be assumed to vary at each small time-interval during each firing round. This justification introduces a way to consider the radiation boundary conditions at the bore and at the outer surfaces of a gun tube. The heat input is permitted to vary with time; in general, the propellant gas temperature function can be assumed to be a combination of various exponential functions.

METHOD OF SOLUTION

Since the boundary condition (3) is not homogeneous, the method of separation of variables cannot be applied directly. The Laplace

transform method is considered; however, the authors find that, if the Laplace transform technique is used alone for solving this problem, the inverse transform is very difficult to obtain. The combination of Laplace transform and separation of variables will be adopted.

Because of the linearity of the problem, the solution to this problem is readily verified and may be written in the form

$$T(r,t) = T_a(r,t) + T_s(r) + T_b(r,t); \qquad a < r < b,\ t \geq 0 \tag{5}$$

where $T_a(r,t)$ satisfies the following equations:

$$\frac{\partial^2 T_a}{\partial r^2} + \frac{1}{r}\frac{\partial T_a}{\partial r} = \frac{1}{\alpha}\frac{\partial T_a}{\partial t}; \qquad a < r < b,\ t > 0 \tag{6}$$

$$K\frac{\partial T_a}{\partial r} - h_1 T_a = -h_1 T_g(t); \qquad r = a,\ t > 0 \tag{7}$$

$$K\frac{\partial T_a}{\partial r} + h_2 T_a = 0; \qquad r = b,\ t > 0 \tag{8}$$

$$T_a(r,t) = 0; \qquad a \leq r \leq b,\ t = 0 \tag{9}$$

$T_s(r)$ satisfies the following equations:

$$\frac{d^2 T_s}{dr^2} + \frac{1}{r}\frac{dT_s}{dr} = 0; \qquad a < r < b \tag{10}$$

$$K\frac{dT_s}{dr} - h_1 T_s = 0; \qquad r = a \tag{11}$$

$$K\frac{dT_s}{dr} + h_2 T_s = h_2 T_A; \qquad r = b \tag{12}$$

and $T_b(r,t)$ satisfies the following equations:

$$\frac{\partial^2 T_b}{\partial r^2} + \frac{1}{r}\frac{\partial T_b}{\partial r} = \frac{1}{\alpha}\frac{\partial T_b}{\partial t}\ ; \qquad a < r < b,\ t = 0 \qquad (13)$$

$$K\frac{\partial T_b}{\partial r} - h_1 T_b = 0\ ; \qquad r = a,\ t > 0 \qquad (14)$$

$$K\frac{\partial T_b}{\partial r} + h_1 T_b = 0\ ; \qquad r = b,\ t > 0 \qquad (15)$$

$$T_b(r,t) = f(r) - T_s(r)\ ; \qquad a \le r \le b,\ t = 0 \qquad (16)$$

The solution for $T_a(r,t)$ is the one that satisfies the homogeneous differential equation (6) with nonhomogeneous boundary condition (7), homogeneous boundary condition (8) and with zero initial temperature distribution (9). The solution for $T_a(r,t)$ will be obtained by use of the method of Laplace transform. The problem involving $T_s(r)$ is one of a steady-state temperature distribution that can be readily solved by the usual technique of solving ordinary differential equations. Once the problem regarding $T_s(r)$ is solved, the problem concerning $T_b(r,t)$ is completely defined and is the one with homogeneous differential equation (13) and boundary and initial conditions (14), (15) and (16). Since the boundary conditions involving $T_b(r,t)$ are homogeneous, $T_b(r,t)$ can be obtained by the method of separation of variables. When $T_a(r,t)$, $T_s(r)$, and $T_b(r,t)$ are obtained, then the complete solution to the problem formulated in the previous section is given by equation (5).

SOLUTION

(A) Solution for $T_a(r,t)$

The Laplace transform of equation (6) is given by

$$\frac{d^2\bar{T}_a}{dr^2} + \frac{1}{r}\frac{d\bar{T}_a}{dr} - q^2\bar{T}_a = 0\ ; \qquad a < r < b \qquad (17)$$

where $q^2 = p/\alpha$ and $\overline{T}_a$ is defined by

$$\overline{T}_a = \int_0^\infty e^{-pt}\, T_a(r,t)dt \tag{18}$$

The complete solution of the Bessel equation (17) is

$$\overline{T}_a = AI_0(qr) + BK_0(qr) \tag{19}$$

where I_0 and K_0 are modified Bessel functions of order zero. The constants A and B are determined so that $\overline{T}_a$ satisfies the Laplace transform of equations (7) and (8), namely

$$K\frac{d\overline{T}_a}{dr} - h_1\overline{T}_a = -h_1\overline{T}_g(p) \;; \qquad r = a \tag{20}$$

$$K\frac{d\overline{T}_a}{dr} + h_2\overline{T}_a = 0 \;; \qquad r = b \tag{21}$$

$$\text{where } \overline{T}_g(p) = \int_0^\infty e^{-pt}\, T_g(t)dt \tag{22}$$

Substituting equation (19) into equations (20) and (21), one obtains

$$A\,[-h_1I_0(qa) + KqI_1(qa)] - B\,[h_2K_0(qa) + KqK_1(qa)] = -h_1\overline{T}_g(p) \tag{23}$$

and

$$A\,[h_2I_0(qb) + KqI_1(qb)] + B\,[h_2K_0(qb) - KqK_1(qb)] = 0 \tag{24}$$

Solving A and B from equations (23) and (24), and substituting into equation (19), one obtains

$$\overline{T}_a = \frac{-\overline{T}_g(p)h_1\{[h_2K_0(qb) - KqK_1(qb)]I_0(qr) - [h_2I_0(qb) + KqI_1(qb)]K_0(qr)\}}{\Delta(p)} \tag{25}$$

where $\Delta(p) = [h_2 I_0(qb) + KqI_1(qb)]\,[h_1 K_0(qa) + KqK_1(qa)]$

$$- [h_1 I_0(qa) - KqI_1(qa)]\,[h_2 K_0(qb) - KqK_1(qb)] \qquad (26)$$

Suppose

$$\bar{T}_1 = \frac{-h_1\{[h_2 K_0(qb) - KqK_1(qb)]I_0(qr) - [h_2 I_0(qb) + KqI_1(qb)]K_0(qr)\}}{\Delta(p)} \qquad (27)$$

Then equation (25) becomes

$$\bar{T}_a = \bar{T}_g \bar{T}_1 \qquad (28)$$

T_1 is now determined by the Inversion Theorem, i.e.,

$$T_1 = \mathcal{L}^{-1}[\bar{T}_1] =$$

$$\frac{-1}{2\pi i}\int_{\gamma-j\infty}^{\gamma+j\infty} \frac{e^{p\tau}\, h_1\{[h_2 K_0(qb) - KqK_1(qb)]I_0(qr) - [h_2 I_0(qb) + KqI_1(qb)]K_0(qr)\}}{\Delta(p)}\,dp \qquad (29)$$

The integrand is a single-valued function of p with simple poles at $p = -\alpha\beta_n^2$, where β_n are the roots of

$$[h_2 J_0(\beta b) - K\beta J_1(\beta b)]\,[h_1 Y_0(\beta a) + K\beta Y_1(\beta a)]$$

$$- [h_1 J_0(\beta a) + K\beta J_1(\beta a)]\,[h_2 Y_0(\beta b) - K\beta Y_1(\beta b)] = 0 \qquad (30)$$

Since $I_0(i\beta_n r) = J_0(\beta_n r)$, $I_1(i\beta_n r) = iJ_1(\beta_n r)$,

$K_0(i\beta_n r) = -\pi/2\ [iJ_0(\beta_n r) + Y_0(\beta_n r)]$, and

$K_1(i\beta_n r) = \pi/2\ [-J_1(\beta_n r) + iY_1(\beta_n r)]$

To find the residue at pole $p = -\alpha\beta_n^2$, one needs

$$\left[\frac{d\Delta(p)}{dp} \right]_{p = -\alpha\beta_n^2} = \left[\frac{1}{2\alpha\lambda} \frac{d\Delta(p)}{d\lambda} \right]_{\lambda = i\beta_n}$$

$$= \Bigg\{ \frac{1}{2\alpha\lambda} \Bigg(a\ [h_1 I_1(\lambda a) + K\lambda I_0(\lambda a)]\ [h_2 K_0(\lambda b) - K\lambda K_1(\lambda b)]$$

$$- b\ [-h_1 I_0(\lambda a) + K\lambda I_1(\lambda a)]\ [h_2 K_1(\lambda b) - K\lambda K_0(\lambda b)]$$

$$- a\ [h_2 I_0(\lambda b) + K\lambda I_1(\lambda b)]\ [h_1 K_1(\lambda a) + K\lambda K_0(\lambda a)]$$

$$+ b\ [h_2 I_1(\lambda b) + K\lambda I_0(\lambda b)]\ [h_1 K_0(\lambda a) + K\lambda K_1(\lambda a)] \Bigg) \Bigg\}_{\lambda = i\beta_n} \tag{31}$$

where equation (26) and the recurrence formulae[7] have been used. To simplify equation (31), when $\lambda = i\beta_n$,

$$\frac{-h_1 I_0(\lambda a) + K\lambda I_1(\lambda a)}{h_2 I_0(\lambda b) + K\lambda I_1(\lambda b)} = \frac{-[h_1 K_0(\lambda a) + K\lambda K_1(\lambda a)]}{h_2 K_0(\lambda b) - K\lambda K_1(\lambda b)} =$$

$$\frac{-[h_1 J_0(\beta_n a) + K\beta_n J_1(\beta_n a)]}{h_2 J_0(\beta_n b) - K\beta_n J_1(\beta_n b)} = \frac{-[h_1 Y_0(\beta_n a) + K\beta_n Y_1(\beta_n a)]}{h_2 Y_0(\beta_n b) - K\beta_n Y_1(\beta_n b)} = \rho \tag{32}$$

With the use of this result and the Wronskian relation, equation (31) can be written

$$\frac{1}{2\alpha}\left[\frac{1}{\lambda}\,\frac{d\Delta(p)}{d\lambda}\right]_{\lambda = i\beta_n} = \frac{1}{2\alpha\beta_n^2}\left[\rho\,(h_2^2+K^2\beta_n^2) - \frac{(h_1^2+K^2\beta_n^2)}{\rho}\right]$$

$$= \frac{(h_2^2+K^2\beta_n^2)[-h_1J_0(\beta_n a)-K\beta_nJ_1(\beta_n a)]^2-(h_1^2+K^2\beta_n^2)[h_2J_0(\beta_n b)-K\beta_nJ_1(\beta_n b)]^2}{2\alpha\beta_n^2[-h_1J_0(\beta_n a)-K\beta_nJ_1(\beta_n a)]\,[h_2J_0(\beta_n b)-K\beta_nJ_1(\beta_n b)]}$$

$$= \frac{F\,(\beta_n)}{2\alpha\beta_n^2[-h_1J_0(\beta_n a)-K\beta_nJ_1(\beta_n a)]\,[h_2J_0(\beta_n b)-K\beta_nJ_1(\beta_n b)]} \tag{33}$$

where

$$F(\beta_n) = (h_2^2+K^2\beta_n^2)\,[h_1J_0(\beta_n a)+K\beta_nJ_1(\beta_n a)]^2-(h_1^2+K^2\beta_n^2)\,[h_2J_0(\beta_n b)-K\beta_nJ_1(\beta_n b)]^2 \tag{34}$$

The residue at the pole $p = -\alpha\beta_n^2$ or $\lambda = i\beta_n$[8] is

$$\lim_{\substack{p \to -\alpha\beta_n^2 \\ \lambda \to i\beta_n}} (p+\alpha\beta_n^2)\; - \frac{e^{pt}\, h_1\{[h_2K_0(\lambda b)-K\lambda K_1(\lambda b)]I_0(\lambda r)-[h_2I_0(\lambda b)+K\lambda I_1(\lambda b)]K_0(\lambda r)\}}{\Delta(p)}$$

$$= \pi\alpha h_1\,\frac{\beta_n^2}{F(\beta_n)}\, e^{-\alpha\beta_n^2 t}\, C_0(\beta_n,r)[h_1J_0(\beta_n a)+K\beta_nJ_1(\beta_n a)][h_2J_0(\beta_n b)-K\beta_nJ_1(\beta_n b)] \tag{35}$$

where

$$C_0(\beta_n,r) = [h_2J_0(\beta_n b)-K\beta_nJ_1(\beta_n b)]Y_0(\beta_n r)-[h_2Y_0(\beta_n b)-K\beta_nY_1(\beta_n b)]J_0(\beta_n r) \tag{36}$$

Thus,

$$T_1(r,t) = \pi\alpha h_1 \sum_{n=1}^{\infty} \frac{\beta_n^2}{F(\beta_n)} e^{-\alpha\beta_n^2 t} C_o(\beta_n,r)\, G(\beta_n) \tag{37}$$

where

$$G(\beta_n) = [h_1 J_o(\beta_n a)+K\beta_n J_1(\beta_n a)][h_2 J_o(\beta_n b)-K\beta_n J_1(\beta_n b)] \tag{38}$$

Applying the Inversion Theorem to equation (28), one obtains

$$T_a(r,t) = \mathcal{L}^{-1}[\overline{T}_a] = \mathcal{L}^{-1}[\overline{T}_g \overline{T}_1] \tag{39}$$

Applying the Convolution Theorem,[8] one notes that

$$T_g(t) = \mathcal{L}^{-1}[\overline{T}_g(p)] \quad \text{and} \quad T_1(r,t) = \mathcal{L}^{-1}[\overline{T}_1(r,p)]$$

And finally

$$T_a(r,t) = \int_0^t T_g(t-\zeta)\, T_1(\zeta)\, d\zeta$$

$$= \pi\alpha h_1 \int_0^t T_g(t-\zeta) \sum_{n=1}^{\infty} \frac{\beta_n^2}{F(\beta_n)} e^{-\alpha\beta_n^2 t} C_o(\beta_n,r)\, G(\beta_n)\, d\zeta$$

$$= \pi\alpha h_1 \sum_{n=1}^{\infty} \frac{\beta_n^2}{F(\beta_n)} C_o(\beta_n,r)\, G(\beta_n) \int_0^t T_g(t-\zeta)\, e^{-\alpha\beta_n^2 \zeta}\, d\zeta \tag{40}$$

(B) Solution for $T_s(r)$

The general solution of the differential equation (10) is

$$T_s(r) = A\log r + B \tag{41}$$

where A and B are constants that are determined so that T_s satisfies the boundary conditions (11) and (12).

Substituting equation (41) into equations (11) and (12) and solving for A and B, one obtains

$$A = -\frac{h_1 h_2 T_A}{K\left(\frac{-h_1}{b} - \frac{h_2}{a}\right) - h_1 h_2 \log\frac{b}{a}} = \frac{h_1 h_2 T_A}{K\left(\frac{h_1}{b} + \frac{h_2}{a}\right) + h_1 h_2 \log\frac{b}{a}}$$

$$B = \frac{\left(K\frac{h_2}{a} - h_1 h_2 \log a\right) T_A}{K\left(\frac{h_1}{b} + \frac{h_2}{a}\right) + h_1 h_2 \log\frac{b}{a}} \tag{42}$$

Substituting A and B into equation (41), one gets

$$T_s(r) = \frac{T_A}{K\left(\frac{h_1}{b} + \frac{h_2}{a}\right) + h_1 h_2 \log\frac{b}{a}}\left[h_1 h_2 \log\frac{r}{a} + K\frac{h_2}{a}\right];\ a < r < b \tag{43}$$

(C) Solution for $T_b(r,t)$

Once $T_s(r)$ has been obtained from equation (43), the initial condition equation (16) involving $T_b(r,t)$ is completely defined. Since the boundary conditions (14) and (15) are homogeneous, the homogeneous differential equation (13) can be readily solved by separation of variables. The function $T_b(r,t) = R(r)\,\theta(t)$ is a solution provided

$$\frac{\theta'}{\alpha\theta} = \frac{1}{R}\left(R'' + \frac{R}{r}\right) \tag{44}$$

Since the member on the left is a function of t alone and that on the right is a function of r alone, they must be equal to a constant, say, $-\beta^2$. Hence, one has equations

$$rR'' + R' + \beta^2 rR = 0 \tag{45}$$

$$\theta' + \alpha\beta^2\theta = 0 \tag{46}$$

The equation in R is Bessel's equation. The complete solution for R can be written

$$R(r) = A\, J_0(\beta r) + B\, Y_0(\beta r) \tag{47}$$

The solution of equation (46) can be written

$$\theta(t) = C\, e^{-\alpha\beta^2 t} \tag{48}$$

where A, B, and C are constants that can be determined from equations (14), (15), and (16). Under equations (14) and (15), one finds that

$$T_b(r,t) = \sum_{n=1}^{\infty} D_n\, e^{-\alpha\beta_n^2 t}\, [h_1 Y_0(\beta_n a) + K\beta_n Y_1(\beta_n a)]\, J_0(\beta_n r) \tag{49}$$

where $\beta = \beta_n$ are the roots of the following equation

$$\begin{aligned}&[h_2 J_0(\beta b) - K\beta J_1(\beta b)]\, [h_1 Y_0(\beta a) + K\beta Y_1(\beta a)] \\ &- [h_1 J_0(\beta a) + K\beta J_1(\beta a)]\, [h_2 Y_0(\beta b) - K\beta Y_1(\beta b)] = 0\end{aligned} \tag{50}$$

This equation can be noted to be identical to equation (30).

At t = 0, using equations (16) and (49), one has

$$f(r) - T_s(r) = \sum_{n=1}^{\infty} D_n\, \overline{C}_0(\beta_n, r) \tag{51}$$

where $\overline{C}_0(\beta_n,r) = [h_1Y_0(\beta_na) + K\beta_nY_1(\beta_na)]\ J_0(\beta_nr)$

$$- [h_1J_0(\beta_na) + K\beta_nJ_1(\beta_na)]\ Y_0(\beta_nr) \qquad (52)$$

The constants D_n can be determined by use of the fact that the set of cylinder functions

$\overline{C}_0(\beta_n,r)$; n = 1, 2, 3, . . .

is an orthogonal set on the interval $a < r < b$. Hence, one obtains

$$D_n = \frac{\int_a^b r\,\overline{C}_0(\beta_n,r)\,(f(r) - T_s(r))\,dr}{\int_a^b r\,\overline{C}_0^2\,(\beta_n,r)\,dr} \qquad (53)$$

In compact form, equation (49) can be written

$$T_b(r,t) = \sum_{n=1}^{\infty} D_n\, e^{-\alpha\beta_n^2t}\,\overline{C}_0(\beta_n,r) \qquad (54)$$

where D_n is defined by equation (53), and $\overline{C}_0(\beta_n,r)$ is defined by equation (52).

SUMMARY

The analytical solution to the problem formulated in a previous section is given by the sum of equations (40), (43), and (53). The gas and the natural convection coefficients h_1 and h_2, respectively, can be assumed to be variable for each time interval. The initial temperature distribution at the beginning of each time interval will be set equal to the final temperature distribution of the previous time interval. If the temperature distribution of a barrel under continuous firing conditions is sought, the final temperature distribution of one firing round will be considered as the initial temperature distribution of the successive firing round.

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13. ABSTRACT

The analytical solution of the heat conduction equation for a gun tube (circular, thick-walled cylinder) assumed, for simplicity, to be of infinite length, is obtained. The temperature function is assumed to have axial and angular symmetry and, therefore, is dependent only upon time and radial distance. The axial flow has been neglected because the projectile moves through the tube at a much faster rate than heat can penetrate the tube. The initial and boundary conditions have been considered as close as possible to those encountered during the repeated firing of a gun. The temperature distribution in a barrel can be determined provided that the propellant gas temperature during the firing of each round is known and that the nature of the heat transfer from the gas to the barrel wall is known.

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